

3.7.3. A STUDY OF GRAVITY-WAVE SPECTRA IN THE TROPOSPHERE AND STRATOSPHERE AT 5-MIN TO 5-DAY PERIODS WITH THE POKER FLAT MST RADAR

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1. INTRODUCTION

The wind field in the middle atmosphere can be decomposed into a continuum of spatial and temporal scales. At short time scales (of the order of 0.1-3 hr) the wind field is dominated by transient buoyancy waves with horizontal wavelengths of a few km to several hundred km. With recent improvements in the MST radar technique, it has become possible to measure, almost continuously, the velocity field at fixed altitudes for periods longer than a month (GAGE and BALSLEY, 1984; LARSEN et al., 1982). Instrumented aircrafts provide similar measurements along flight paths that are at least a few thousand km long (see e.g. LILLY and PETERSEN, 1983; NASTROM and GAGE, 1983).

Kinetic-energy spectra of the horizontal wind field obtained from several sets of such measurements show distinct power-law behaviors with frequency and wave number over wide range of scales extending to several days in time and several thousand km in the horizontal. A simple and direct scaling between the frequency and wave-number spectra exists under the Taylor Hypothesis, that the perturbations move with the mean horizontal wind without deformation (in a statistical sense). This makes possible first-order comparisons of frequency and wave-number spectra.

A mesoscale power-law behavior, $S(f) \sim f^{+n}$ or $S(k) \sim k^{+n}$ of the frequency (f) and the horizontal wave-number (k) spectra (S) has been reported in several studies with values of the spectral index (or slope) n in the vicinity of $-5/3$ (see e.g. GAGE, 1979; BALSLEY and CARTER 1982; NASTROM and GAGE, 1983, 1985). It has been suggested that the associated horizontal velocity perturbations in the atmosphere are either a manifestation of two-dimensional turbulence (GAGE and NASTROM, 1984) or of a universal cascade of buoyancy waves (VANZANDT, 1982).

In this paper, we present an analysis of frequency spectra at periods of about 5 days to 5 min from two 20-day sets of velocity measurements in the stratosphere and troposphere (ST) region obtained with the Poker Flat MST radar during January and June, 1984.

In Section 2, we outline a technique based on median filtering and averaged order statistics for automatic editing, smoothing and spectral analysis of velocity time series contaminated with spurious data points or outliers. The validity of this technique and its effects on the inferred spectral index has been tested through simulation. Spectra obtained with this technique are discussed in Section 3. The measured spectral indices show variability with season and height, especially across the tropopause. The discussion in Section 4 briefly outlines the need for obtaining better

climatologies of velocity spectra and for refinements of the existing theories to explain their behavior.

2. MEDIAN FILTERING, EDITING, SMOOTHING, AND SPECTRAL ANALYSIS OF VELOCITIES

MST radar provide measurements of radial velocity at selected ranges from spectral moments of the scattered signal. Spectra of the scattered signals are often contaminated with external interference, which produces spurious estimates of radial velocities. These spurious velocity values often occur as large, single or multiple spikes. Unless the spurious values, called outliers, can be successfully detected and removed, they would tend to introduce a whitening of high-frequency components in the velocity spectra. The velocity spectra are commonly obtained through discrete Fourier transform (DFT) of the uniformly sampled velocity time series using the fast Fourier transform (FFT) algorithm.

The spectrum $S(f)$ is usually approximated by the time-averaged periodogram $[P(f)]$, which is the squared magnitude of the Fourier transform. Periodogram provides only a distorted estimate of the spectrum. This distortion is most severe in the presence of trends or a slowly varying mean value component and arises due to aliasing of the autocorrelation function. Presence of a trend in the velocity data is likely to produce spectra with an inverse-square frequency dependence, or a spectral index $n = -2$. The simplest way to remove trends is to subtract the dc value from each segment of the velocity time series before spectral analysis. This has been done for the analysis described in this paper.

The spectral index (n) is obtained by a least-square error fit or regression of the form: $\log S(f) = n \log f + c$, where c is a constant. Equi-spaced frequency points tend to become clustered at high frequencies along the $\log f$ axis. This tends to bias n towards the high-frequency end, where the effect of outliers is likely to be most serious. To minimize this bias, it is necessary (i) to implement a scheme for locating outliers, and interpolating or smoothing through their locations and gaps; and (ii) to obtain the spectra of smoothed velocity time series in several bands of approximately equal width on the $\log f$ axis, with a proportionate number of points in each band.

Figure 1(a) shows the radial velocity time series at 10.3 km height for an off-vertical antenna. This series was sampled at 1-min intervals over 16.3 hr or 1000 min. Many outliers can be seen distinctly since they clearly do not belong with the rest of the data. If the velocity changes associated with the outliers are real, they would imply accelerations that cannot be justified on physical grounds. There also was a gap of about 5.5 hr near the middle. Figure 1(b) shows a filtered and interpolated time series that has been smoothed through a 30-min window, at 15-min intervals. The filtering and interpolation procedure is briefly outlined below.

Since some smoothing of time series is desirable, we prefer to look at all the N points within a smoothing window collectively. Suppose these points are sorted by their numerical values in an ascending or descending sequence. Outliers occur mostly near the top and the bottom of this sequence, whereas most of the valid data points tend to be clustered about the median. Median is a robust indicator of location with respect to outliers. If we reject $x\%$ of the highest and $x\%$ of the lowest points, and average over the remaining then this average will be almost unaffected by outliers. It is not necessary to sort (or order) all the N points. Sorting is needed for only $x\%$ of the highest and $x\%$ of the lowest values. Two types of errors can occur in this averaging scheme depending on N and x : a valid data point may be mistaken for an outlier, and an outlier may be mistaken as a valid data point. These errors are analogous to those encountered in transmission of signals over noisy

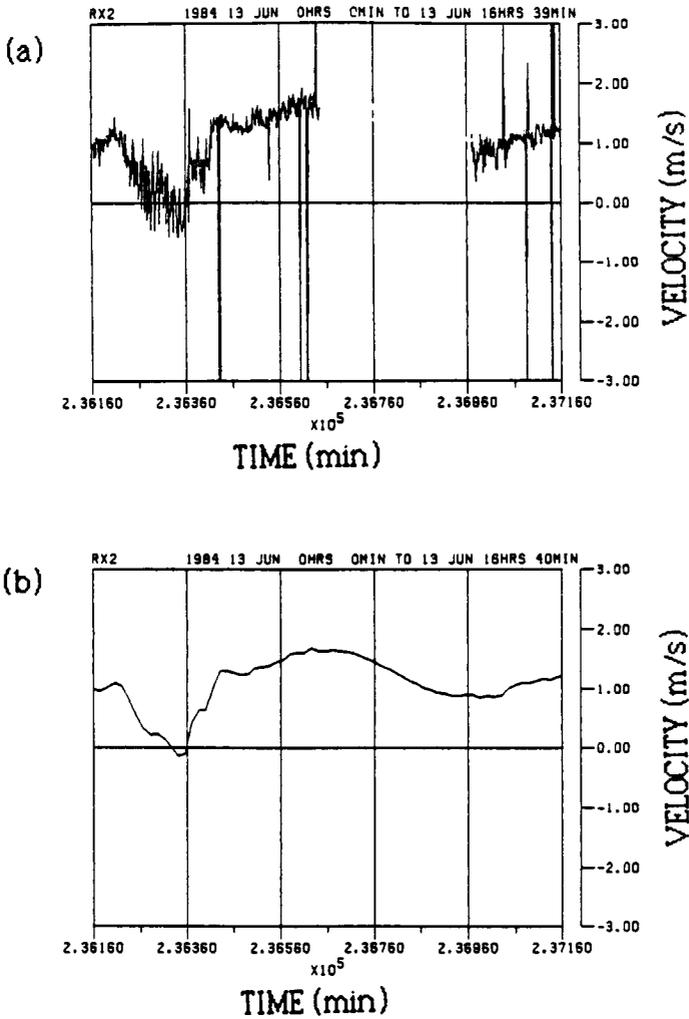


Figure 1. (a) Unedited time series of radial velocity for a 1000-min period observed with the Poker Flat MST radar, showing outliers and a 5.5-hr data gap. (b) Same time series after 30-min median filtering, smoothing, and interpolation with cubic splines. (Data for receiver 2, 13 June, 1984, 00 hr 00 min to 16 hr 39 min; Height 10.33 km.)

communication channels. Prior statistical information about data points and outliers (if available) can be used to minimize these errors. An alternative approach is to experiment with different values of rejection levels x . In the analysis of Poker Flat data we have found that for $N = 30$, rejection levels $x = 7.5, 10$ and 20% did not produce a significant effect on smoothed averages. A rejection level of 10% was used in subsequent analyses. For small values of $N (< 10)$ smoothing is not desirable, but median still provides reasonable outlier rejection. In many of the cases examined, the outlier-rejection schemes based on median and order statistics, perform as well as a human editor.

A few long gaps can be filled by interpolation of data smoothed through a median filter discussed above. We have used Cubic Splines (DEBOOR, 1978) as interpolating functions. To avoid instabilities in interpolation, it is recommended that the average slopes at the end points of each gap should be calculated separately and used to constrain the interpolated results. Most stable results are obtained by setting this slope to zero, e.g., by duplicating the end points in the gap.

Figure 2 shows a composite spectrum of meridional wind obtained from radial velocities measured along three pointing directions, one of which is close to vertical and two are 15 deg off vertical in two orthogonal planes. The spectrum was obtained from 20-day long, almost continuous velocity measurements with a 1-min time resolution. It covers a frequency range of over 3 decades for periods corresponding to 5 days-5 min and was obtained separately over three overlapping bands, each over a decade wide.

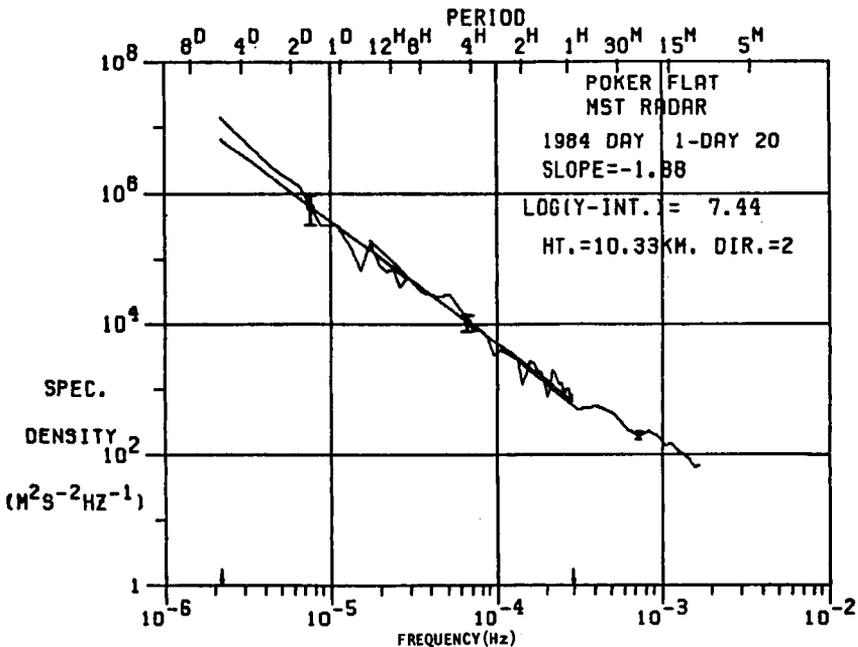


Figure 2. A composite spectrum of meridional winds obtained in three overlapping bands, each about 1 decade wide. Arrows show the frequency range (periods about 1 hr to 5 days) over which a spectral index was obtained through linear regression. Uncertainties in the spectrum estimates are also shown. (Data for January 1 to January 20, 1984.)

For the low and intermediate frequency bands, first the radial velocities were subjected to median filtering through a 30-min running window at 15-min intervals. The radial velocities were further smoothed for each frequency band, and decomposed into the three orthogonal velocity components: zonal, meridional and vertical. This decomposition is possible only for velocities smoothed over time scales of 1 hr or more. The spectrum was then computed as the averaged periodogram over each band. The number of periodograms averaged was 4 for the low and 15 for the intermediate frequency band.

For the high frequency band, the radial velocities cannot be resolved into orthogonal components due to the presence of short-period gravity waves. It is possible, however, to combine and rescale the spectra of radial velocities to obtain the spectra of the orthogonal components. The radial velocities were screened through a 5-point running median filter, with the output sampled at 2.5-min intervals. No attempt was made to interpolate through gaps. Approximately 80-90 periodograms obtained from short continuous segments were averaged over the entire 20-day period. The averaged periodograms of radial velocities were combined and rescaled to obtain the spectra of orthogonal velocity components.

The uncertainties in the estimation of the composite spectra are shown by the error bars for each frequency band (see e.g., BLACKMAN and TUKEY, 1958 or OPPENHEIM and SCHAFER, 1975) at one standard deviation level. The uncertainty is smallest at the highest frequencies due to the larger number of periodograms averaged in this band. In conventional methods of spectral analysis all the frequency points are equispaced, and the uncertainty in the spectrum magnitude is uniform throughout. The uncertainty in estimating the spectral index (n) through exponential regression (linear regression of $\log S(f)$ versus $\log f$) is described in statistical texts (e.g., BROWNLEE, 1965). Through the use of composite spectra the uncertainty in spectral magnitude is considerably reduced at higher frequencies. Within an uncertainty of one standard deviation in spectral magnitude, composite spectra provide estimates of spectral index to within ± 0.1 over three decades and to within ± 0.15 over two decades. The composite spectra shown in Figure 2, show a distinct power-law behavior over three decades of frequency. In this example the spectral index obtained by regression (at 5 day - 1 hr periods) was -1.88. We have, however, tried to obtain additional confidence in the performance of analysis methods outlined above as follows.

The spectra and spectral slopes were also independently estimated by the correlation method. The results obtained by the two methods were almost identical and did not show any systematic differences. This indicates the effectiveness of trend removal used in the periodogram method in reducing distortions of spectra.

As an alternative check, time series for spectra with known spectral indices were synthesized as the sum of a large number of sinusoids in random phase. To these, random outlier values and data gaps were added with about the same statistics as in the original data. Median filtering and spectrum analysis of the synthesized time series recovered the spectral index to within 5% to 10% in all cases considered. We, therefore, conclude that the departures of spectral index from the $5/3$ rd shape as shown for example in Figure 2, are significant.

3. VARIATION OF SPECTRAL INDEX WITH HEIGHT AND SEASON

The composite spectra for several ST heights (8 - 23 km at 2.2 km step) and for all the three orthogonal velocity components are shown for 20 days of January, 1984 data in Figure 3. For clarity, the spectra for different heights

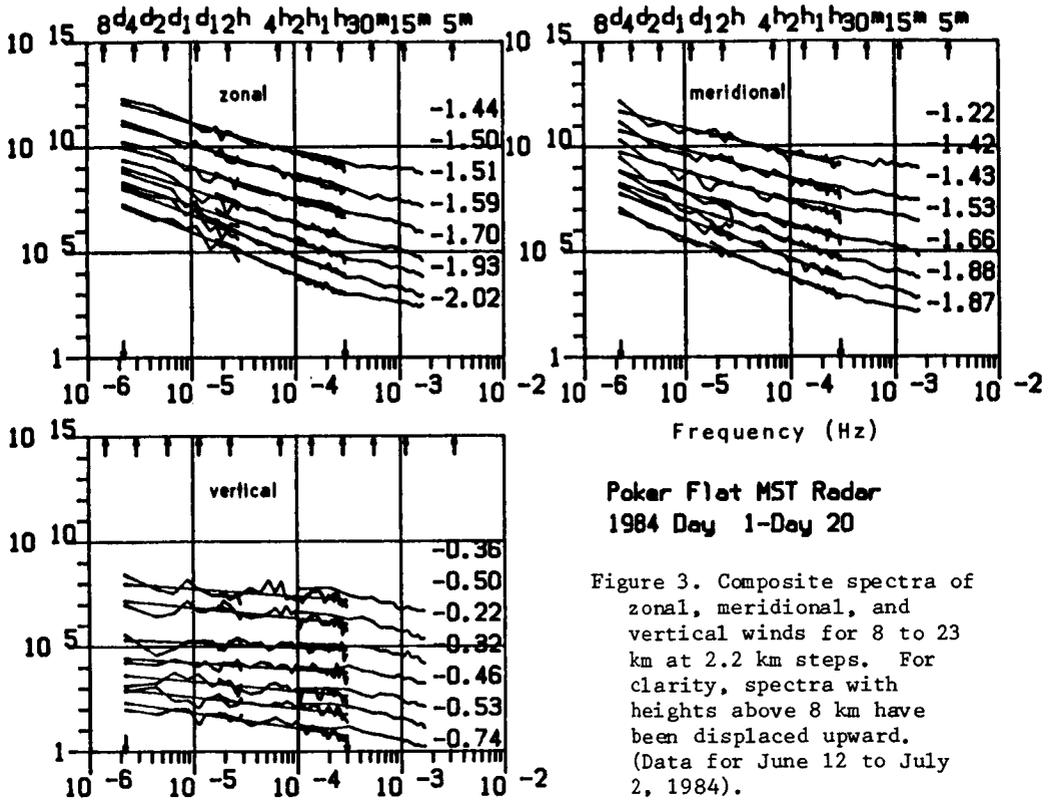


Figure 3. Composite spectra of zonal, meridional, and vertical winds for 8 to 23 km at 2.2 km steps. For clarity, spectra with heights above 8 km have been displaced upward. (Data for June 12 to July 2, 1984).

have been displaced upward by a constant amount. The behavior of zonal and meridional spectra is similar, with a power-law behavior extending to periods below 1 hr. The corresponding spectra for the vertical velocities are almost flat, with a hump between 3 hr and 24 hr at some altitudes. The spectral index for each height is also given. There is a weak indication of the presence of a semidiurnal (12 hr) tide in the spectra of the horizontal velocity components (see Figure 3, zonal).

Figure 4(a) shows the profiles of spectral index for each component for January, 1984. Figure 4(b) is a similar plot for June, 1984. The spectral index of the horizontal components reaches a minimum value of about -2 near the winter tropopause. A similar behavior is seen for June, 1984, except that the minimum value of spectral slope is about -1.4. The behavior of zonal and meridional spectral slopes is similar. The slope of zonal spectra appears, however, to be slightly steeper than that of the meridional spectra; the difference in their slopes is more pronounced during summer. The spectral index of vertical velocity is usually -0.5 to -1.0. Its height variation is much more erratic during winter.

Therefore, we conclude that the spectral index of about $-5/3$ for the horizontal velocities is exceptional. Indeed, the only case when it is close to this value is for 12-km altitude during January, just above the winter tropopause.

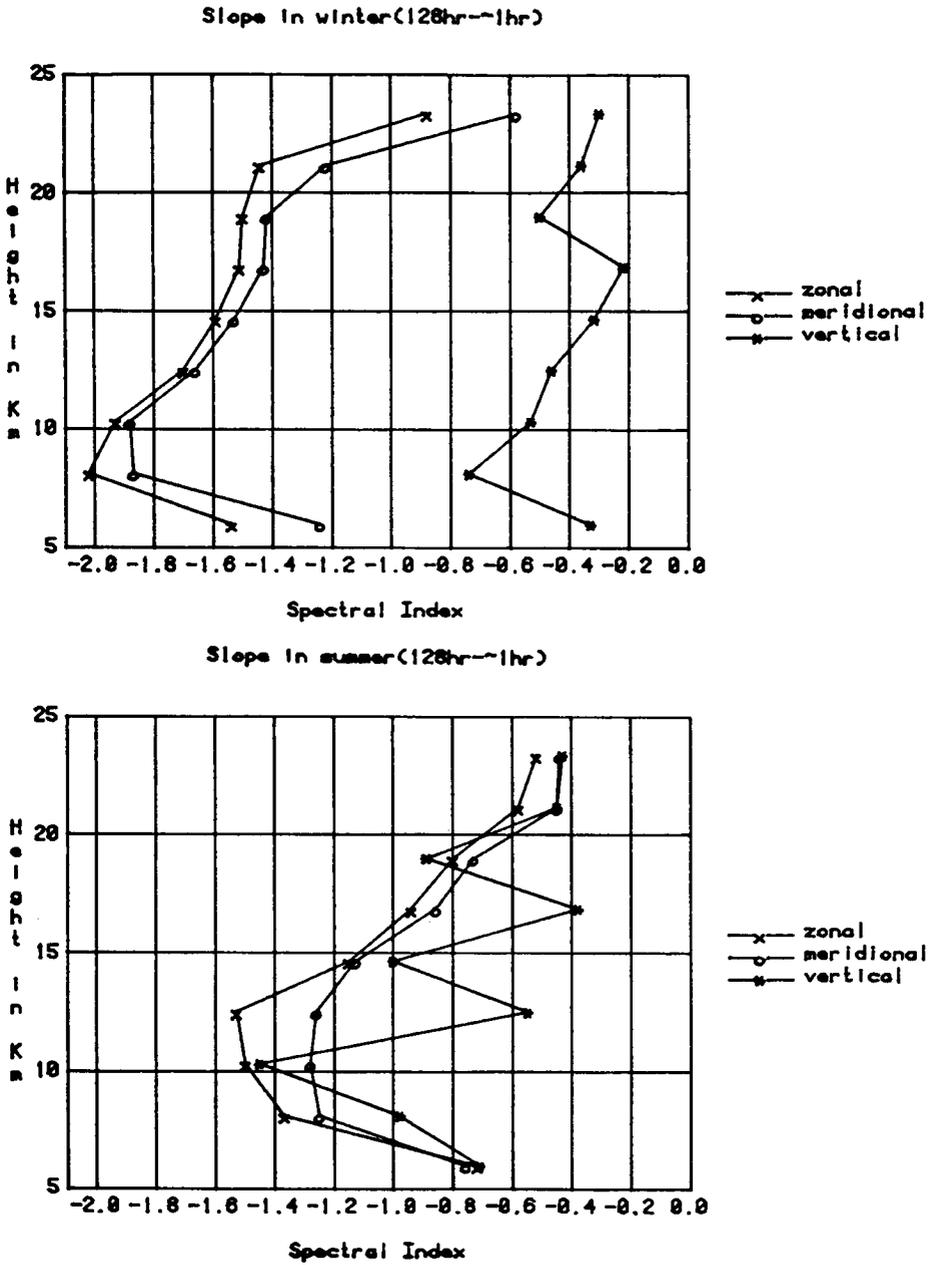


Figure 4. Spectral index of zonal, meridional, and vertical winds for 1-128 hr periods (a) January, 1984, and (b) June, 1984.

4. DISCUSSION

The analysis of velocity spectra described above shows that the spectral index of the horizontal velocity components varies with altitude and season. The spectral index tends to approach a minimum value in the vicinity of the tropopause. The altitude variation of spectral index is similar in winter and summer, except that the indices have higher values (less negative) in the summer. There does not seem to be a preferred tendency for a $-5/3$ spectral slope, though this slope does occur in the 15-20 km range during winter. The variability of spectral index of horizontal velocities is also evident in the Poker Flat ST data analyzed by LARSEN et al. (1982). On the basis of the two-dimensional turbulence hypothesis, a spectral index of $-5/3$ is predicted for horizontal velocity spectra (GAGE, 1979). We conclude that the observations of spectral index reported here do not provide sufficient evidence for this hypothesis.

The time-scales considered in this analysis range from about 5 days to 1 hr. For a mean horizontal wind of 10 m/s that is typical for winter, the corresponding horizontal scales are 4500 km to 36 km. Mean winds during the summer are only about 2.5 m/s and the horizontal scales are smaller by a factor of 4. The steep slopes (-3) that have been reported in aircraft measurements for scales longer than about 1000 km (NASTROM and GAGE, 1985) have not been discerned in the spectra reported here. It is conceivable that the assumptions under which this simple scaling is possible are not valid at scales much longer than a day and 1000 km.

The spectral index of vertical velocities shown in Figure 4 is typically about -0.5 during winter. The average index is closer to -1 during summer. In both seasons, this index shows a large variability with values ranging from -1.4 to -0.2 . LARSEN et al. (1985) have reported similar values of spectral index of vertical velocities measured at Arecibo with an average value of -1 . Vertical wave number spectra obtained by KUO et al. (1985) using high-resolution data from the SOUSY radar show a similar behavior, with a distinct variation across the tropopause. According to LARSEN et al. (1985) an index of the order of $1/3$ should be expected for a universal spectrum of buoyancy waves along the vertical direction provided that the horizontal spectra exhibit a $-5/3$ slope.

The universal buoyancy wave spectrum hypothesis (VANZANDT, 1982) attempts to seek relations between spectra of velocity components as functions of vertical and horizontal wave number. Though our observations do not appear to provide conclusive evidence for this hypothesis, they certainly suggest the need for a better climatology of horizontal and vertical velocity spectra and their comparison with improved models. SCHEFFLER and LIU (1985) and VANZANDT (1985) have proposed models for acoustic-gravity wave spectra of radial velocities, Doppler shifted by fluctuations in the background wind. There also is a need for improved radar experiments to provide measurements for valid comparisons with theoretical models.

Finally, it should be stressed that mesoscale spectra can possibly be influenced by several mechanisms viz. two-dimensional turbulence, a universal behavior attributed to buoyancy waves, and Doppler shifts of buoyancy waves through variable background winds. A clear-cut distinction between these mechanisms may not be readily possible, unless additional measurable parameters are introduced in theoretical models.

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